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Interim Report -- Oxyance -- A New Measure of Edge Sharpness

by

[redacted]

INTRODUCTION

In evaluating the quality of a photographic image, criteria must be based upon the intended end use of that image. If it is primarily intended to measure the image with an electro-optical or optical mechanical system, criteria should be based primarily on the information content in the image. Here the limitation of information content may be established by resolving power and granularity. If the intended use involves visual examination another set of quality criteria may apply. It has been shown that visual interpretation is largely concerned with relatively coarse detail and that ultimate resolving power is normally beyond the range of vision even when assisted by moderately low power magnification (in this category, anything up to about 12X may be considered). Visual recognition of such coarse detail has been shown to be largely influenced by what has been called the sharpness of the image. This is not necessarily the same condition which prevails for maximum resolution and which may in fact be largely independent of resolution. Sharpness seems to be a function of the photographic edges and several attempts have been made to express the edge sharpness as a single parameter. Among those that have been used in the past have been the average gradient; that is, the average rate of change of density with distance across the edge; the maximum gradient; that is, the maximum slope of a density-distance trace across the edge, and acutance, a parameter devised by [redacted], based on the mean square density gradient. This is the edge parameter which has been included in the Image Quality Meter. We now propose a new measure of edge sharpness which we are designating as oxyance (from the Greek root for sharp, "oxus"), based on the area between the edge trace and a vertical line through the trace.

This measure is intended to eliminate some of the disadvantages found when the automatic computation of acutance was included in the [redacted] Image Quality Meter. Oxyance, as compared to acutance, will be largely independent of grain noise, and should be virtually independent of the points at which a given edge is crossed. In automatic computation, it will be based on direct integration of voltages, eliminating any differentiation. Consequently, the computer will be simpler, more reliable, and less subject to noise produced anomalies. In addition, acutance is expressed in units of mm^{-1} and is unbounded, while oxyance is a dimensionless number, ranging from 0 to 1.

Experiments have been conducted with all of the above parameters seeking to correlate the measure of edge sharpness with subjective evaluation of sharpness in pictorial material. Of the first three, only acutance has shown an acceptable degree of correlation, and the fourth, oxyance, is currently being investigated in a series of psycho-physical experiments to determine whether its correlations are equal or superior to acutance.

EDGE PARAMETERS

A photographic edge may be defined as existing if it represents a boundary condition between two adjacent areas such that:

a) the density of area A differs from the density of area B by more than the detectable limit. (In cases of visual observation this would be more than .02 density units, which is about the threshold of detectable differences for the human eye under moderately bright viewing conditions) and

b) a line between given points in area A and area B the change of density must be greater than the minimum rate of change perceptible under the viewing conditions being used. If an edge exists it can be represented by a microdensitometer trace (see diagram, Figure 1). That is, by traversing the photographic material across the slit of the microdensitometer and obtaining the log-luminance response as a function of the distance and by adjusting the direction of travel so as to produce a maximum curve (that is, essentially perpendicular to the edge) we have a graphic portrayal of the data upon which all measures of edge sharpness have been based.

The slide shows edge traces for two edges - both having the same density difference and the same length of density transition. The average gradient is, of course, the same for both traces, and the maximum gradient may or may not be the same. The subjective evaluation of the sharpness of the edges represented by the edges would be different, and this difference would be indicated by the measured differences in either acutance or oxyance.

ACUTANCE

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[redacted] based their edge sharpness measure on the concept that eye judges sharpness by scanning across the edge, responding to small luminance gradients. For the means of averaging the series of infinitesimal density gradients, they chose the mean-square of the density gradients. (This criterion weights larger density gradients more heavily than smaller ones, which probably approximate the physiological situation; a simple mean-gradient evaluation would not do this.) The mean square density gradient can be shown to be:

$$\frac{\int_A^B \left(\frac{\Delta D}{\Delta x}\right)^2 dx}{X_B - X_A}$$

in which the points A, B and the distances and density differences (ΔD) are as shown in Figure 1.

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The value of the limiting gradients A and B which establish the integration limits are very important to the accuracy of the calculation. [redacted]

[redacted] suggested that the human threshold-gradient-sensitivity, at 4X optical magnification, 0.005 density units per micron, should be used.

Finally, it was felt that the subjective evaluation of edge sharpness was influenced by the total density difference as well as the mean-square gradient. [redacted] original formulation suggested that acutance is equal to the product of the mean-square gradient and the total density scale. This was reinvestigated, however, and it was found that it is preferable to normalize the value of acutance by dividing the mean-square gradient by the total density difference $(D_B - D_A)$; therefore, acutance is currently defined as:

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$$\text{Acutance} = \frac{\int_A^B \left(\frac{\Delta D}{\Delta x}\right)^2 \quad dx}{(X_B - X_A)\ (D_B - D_A)}$$

The fact that the parameter acutance tends to characterize the edge response has been proved experimentally.

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Slide 2 shows a block diagram of the acutance computer section of [redacted] Image Quality Meter. Although it produces good agreement on the average with values assigned to calibrated edges made by [redacted] there is wide spread in individual measurements on a given edge.

OXYANCE

As in the case of acutance, oxyance is also computed from the microdensitometer trace of an "edge" crossing, but instead of using the gradients of the slope of this trace, oxyance depends on a measure of the area between the actual trace and an ideal discontinuous step between the two density levels.

Mathematically it may be defined as follows:

$$N = 1 - \frac{\int_0^1 \left| D_i - D_a \right| \quad dx}{\int_0^1 \left| D_i - D_w \right| \quad dx}$$

where

N = Oxyance

$D_i(x)$ = the ideal density function of a knife edge

b = the value of x at the border between the dark and light portions of the image, assumed always to be that value of x such that $D_a(b) = \overline{D}$; if there is more than one value of x for which $D_a = \overline{D}$, we must arbitrarily choose one of those values of x to be b

$D_a(x)$ = the actual density function of the image under study

D_B = the density attained over the dark portion of the image

D_A = the density attained over the light portion of the image

$D_w(x)$ = the worst possible density function

= $\overline{D}$ for all x

$\overline{D}$ = average density = $.5(D_A + D_B)$

x = the distance horizontally along the negative in dimensionless units such that x = 0 at the left edge and x = 1 at the right edge

Thus 1 - N is the ratio of the area between the actual density curve and the ideal density curve to the area between the ideal curve and the theoretical curve representing zero rate of change of density.

This latter area will always be $\overline{D} - D_A$ so that we can simplify the the definition of N to the following:

$$N = 1 - \frac{\int_0^1 \left| D_i - D_a \right| dx}{\overline{D} - D_A} = 1 - \frac{\int_0^b (D_a - D_A)dx + \int_b^1 (D_B - D_a)dx}{\overline{D} - D_A}$$

While oxyance, as so defined, shows no one to one correspondence to acutance, it has been shown for sets of synthetic edges to correlate closely to acutance. It offers the advantages of being bounded between 0 and 1.0 for the theoretical worst and theoretically ideal edges and to be essentially independent of start and stop positions for computation. In addition to these, it is almost independent of film granularity. Electrical computation of oxyance will be a simpler, relatively noise-independent procedure involving primarily integration. Slide No. 3 shows a typical block diagram for the proposed oxyance computation in the image quality meter.

DEFINITION

Edge response, whether oxyance or acutance, characterizes only the low spatial-frequency response of the system, when the viewing magnification is very high, the important-detail size approaches the limit of resolution. Here, the high spatial-frequency response becomes predominant. An empirical mathematical relationship between acutance, resolving power (R), and viewing magnification, which correlates with the subjective ranking of definition has been established as

$$\text{Definition} = \text{Acutance}\left[1 - \exp(-k_m R^2)\right]$$

where k_m is a constant that depends on the viewing conditions, particularly the viewing magnification. (k_m is equal to 0.007 for normal close-up viewing with the average unaided eye.)

$$\text{Definition} = \exp - (N + k_m R^2)$$

similarly can be used when oxyance is used as the measure of edge sharpness.

(Both oxyance and acutance are symmetrical measures, in that there would be no difference in the magnitude if dense and light areas were interchanged. There is some possibility that subjective evaluation of an edge's sharpness is, on the other hand, not completely independent of relative shape of high and low density portions of its trace. An objective manner for measuring this has not yet been established, but it may be a byproduct of further analysis of the areas used in measuring oxyance and attempts to evaluate such will be included in further extensions of the psychophysical experiments now under way

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MATHEMATICAL AND PSYCHOPHYSICAL TESTS OF OXYANCE

The expression for oxyance used above is one of several possible which use the oxyance area as the criterion for edge sharpness. These are being tested for convenience of electronic implementation, and most meaningful distribution of numerical value for types edges actually met in practice in the range 0 to 1 (0 to 100%). Two sets of tests are being used. One set establishes the relationship between acutance and oxyance for synthetic edges -- that is arbitrary curves which could represent typical photographic edges. The second, far more meaningful, is a psychophysical experiment seeking to establish the correlation between subjective evaluation of the sharpness in pictorial material with the computed oxyance for a reference edge on the same transparency.

SYNTHETIC EDGES

Since the work of [redacted] and others have STATINTL shown a high degree of correlation between subjective sharpness judgment and acutance, it is necessary that any other measure of edge-sharpness show a general correlation to acutance.

The curves in slide No. 5 show this relationship between oxyance and acutance for:

1) a set of synthetic edges consisting of straight line segments

2) a set of synthetic edges based on a 7th degree polynomial STATINTL

3) a set of reference edges calibrated by [redacted] with measurements of oxyance based on microdensitometer traces made on the [redacted] Image Quality Meter

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4) a set of sine waves used as a test pattern for the acutance computer section of the Image Quality Meter

It will be noted that the oxyance-acutance relationship lies in a narrow zone. The differences appear to depend upon the toe and shoulder conditions of a given edge.

While there is no one to one relationship between oxyance and acutance, there is a complex relationship which may be expressed as follows:

If the edge trace is considered as the representation of some function D and if D admits a Maclaurin expansion, (and since it is continuous with all derivatives existing this is a reasonable assumption) then

$$D = \sum_{i=0}^{\infty} a_i x^i$$

Then oxyance $N = 2 \sum_{i=0}^{\infty} a_i \frac{2^i - 1}{i+1}$

and acutance $A = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{ij a_i a_j}{i+j-1}$

For the synthetic edge, assume

a 7th degree polynomial

$$D = \sum_{i=0}^{7} a_i x^i$$

is a good approximation of D

Further let us require that

at $x = 0$ $D = 0$ and $D' = 0$

at $x = 1/2$ $D = 1/2$ and $D'' = 0$

at $x = 1$ $D = 1$ and $D' = 0$

then

$$D = 3(1 + 3/4a)x^2 - (2 + \frac{17}{2}a)x^3 + \frac{35}{4}ax^4 - \frac{7}{2}ax^6 + ax^7$$

where a is the coefficient of x^7

By varying a we can change the magnitude of the slope at the point (1/2, 1/2) of steepest ascent, and directly calculate sets of values for oxyance and acutance.

THE PSYCHOPHYSICAL EXPERIMENT

The first set of comparisons between subjective evaluation of edge sharpness and oxyance are being subjected to statistical analysis. Results are not yet complete but some trends can be indicated.

For the purposes of this experiment, a set of 20 positive transparencies were created from a single negative. This set represented 5 degrees of edge sharpness conditions, prepared on 4 differing photographic emulsions. By using a special contact printing technique with control of the transfer function it was possible to vary the edge-sharpness of the positives. Each transparency also carried a series of reference edges for the measurement of oxyance and acutance.

These transparencies were ranked for sharpness by having them viewed under 4X magnification by a large number of viewers. Individual transparencies were paired at random and the sharper of each pair was designated. 180 sets of 10 each of such comparisons were made and formed the basis of establishing a numerical value for the sharpness rank of each.

The Image Quality Meter was used for the objective measurement of edge sharpness. The acutance for the reference edge of each transparency was automatically computed and simultaneously photographs were made of the microdensitometer displays of their edge traces. The oxyance was computed from planimeter measurements of the oxyance area for each trace.

Preliminary results of comparisons are indicated in the curves shown in the following slides. Further evaluations are being made.

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In addition, investigation will also be made of the effects of edge-peaking or border effects on subjective evaluation of sharpness and its relation to measured oxyance. Further, as was pointed out by [redacted] in a recent communication, the shapes of toe and shoulder may have different weighted influence on subjective evaluation of sharpness. The means for evaluating this factor may be present in our data, but it is expected that additional testing will be required. If this proves to be the case, it will be feasible to modify the formulation and computation of oxyance to take this factor into consideration.

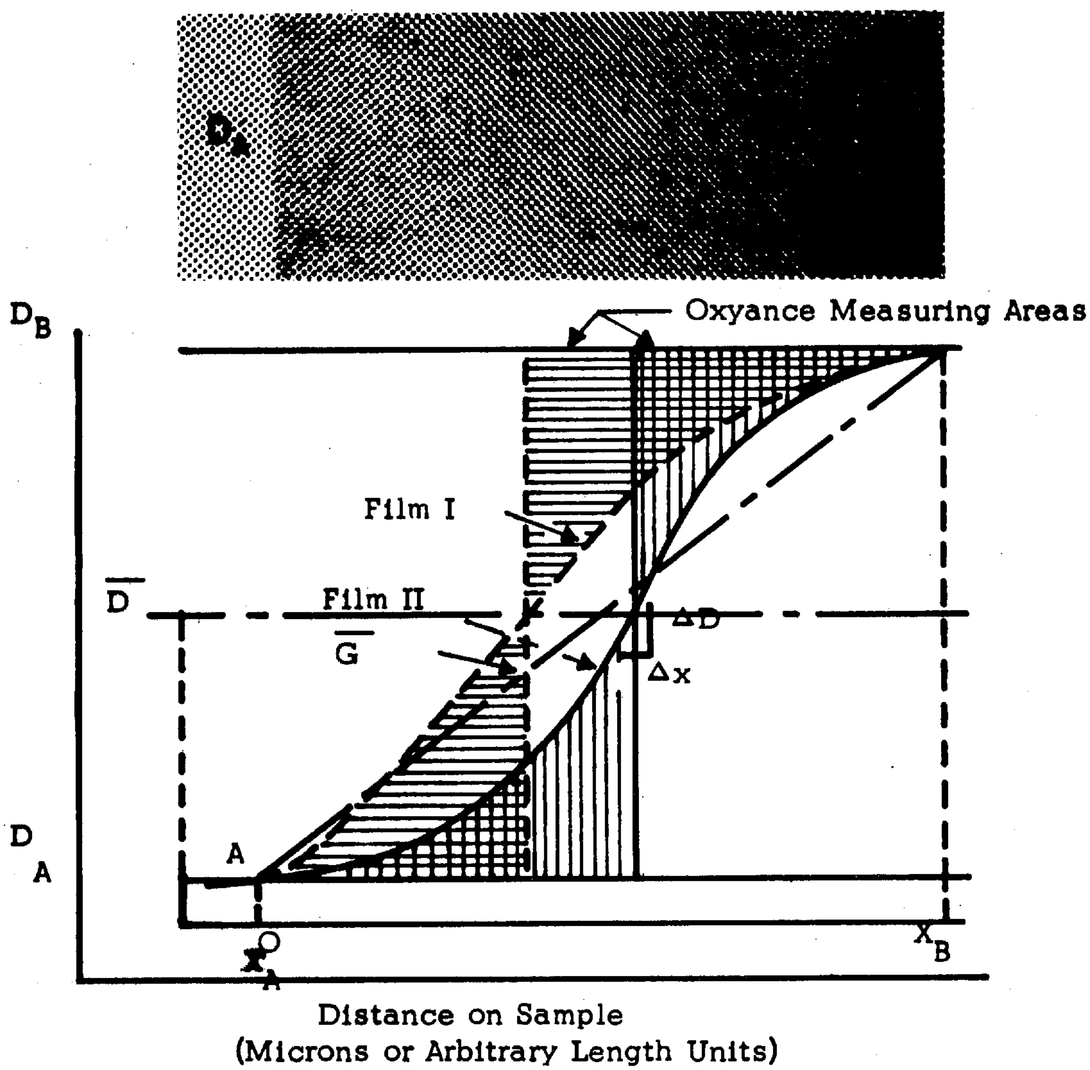

Figure 1. A Photographic Edge and Microdensitometer Trace.